Modelling of Silicon Electrostatic Ultrasonic Transducers

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Techniques of Informatics and Microelectronics for Computer Architecture
MODELLING OF SILICON ELECTROSTATIC ULTRASONIC TRANSDUCERS

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Electrostatic (Capacitive) Transducer

- Generation and Reception of Sound
- Most Critical Component

\[
\begin{align*}
\text{h} & \quad \text{distance between the electrodes} \\
\text{S} & \quad \text{electrode surface} \\
p & \quad \text{acoustic pressure} \\
\text{U} & \quad \text{electrical voltage} \\
\text{U}_0 & \quad \text{polarization voltage}
\end{align*}
\]
Analogue Model

General Form
(Transmitter)
Electrical Part

Capacity between the Electrodes:

\[ C_0 = \frac{\varepsilon_0 S}{h} \]

Electro – Mechanical Transformer :

\[ n = \frac{C U_0}{h} \]

\( U_0 \) … polarization voltage
Mechanical Part

Mechanical Impedance of the Vibrating Disc \((Z_{\text{mM}})\) can be described by

- reduced mass \((M_{\text{mM}})\)
- reduced stiffness \((S_{\text{mM}})\)

In simple cases can be defined analytically, otherwise can be defined by Finite Elements Analysis.

- mechanical resistance due to internal friction

  Can be neglected comparing to the viscous damping in the acoustical part.
Acoustic Part : Front Load

Acoustic Impedance $Z_{aR}$

Radiation Impedance:

for $ka >> 1$ ($f >> c/(2\pi a)$) real part dominates … $R_{aR} = \rho_0 c/S$
Acoustic Part: Back Load

Acoustic Impedance $Z_{aG}$

Impedance of the Air-Gap

Circular Plate with an Air-Gap Open at the Circumference Side

(piston – like movement)

Equivalent Circuit

\[ S = \pi r^2 \]

\[ S_h = 2\pi rh \]

\[ v_z \]

\[ h \]

\[ v_r \]

\[ F_z \downarrow \]

\[ C \]

\[ S : S_h \]

\[ X_1 \]

\[ R \]

\[ V_r \]

\[ F_r \]
Acoustic Part: Back Load

Elements of the Equivalent Circuit (acoustic)

\( C_a \) … air-gap compliance

\[
C_a = \frac{h\pi r^2}{\rho_0 c^2}
\]

\( X_{a1} \) … imaginary part of the impedance in the radial direction

\[
X_{a1} = \frac{2\rho_0 \rho h}{\pi r^3} \left( -\frac{J_0(kr)}{J_1(kr)} + \frac{2}{kr} \right)
\]

\( R_a \) … damping resistance

\[
R_a = \frac{3\mu}{2\pi h^3}
\]

\( \mu \) … dynamic viscosity

for \( kr < 1 \):

\[
X_{a1} = \frac{\omega \rho_0 h}{2\pi r^2}
\]

\[
X_{a1} = \omega M_a
\]
Acoustic Part: Back Load

Circular Plate with an Air-Gap Open at the Center (piston)

\[ S = \pi (r^2 - r_0^2) \]

\[ S_0 = \pi r_0^2 \]

\[ S_h = 2\pi r_0 h \]
Acoustic Part: Back Load

Equivalent Circuit
Acoustic Part : Back Load

Elements of the Equivalent Circuit (acoustic)

\[ C_a = \frac{\pi r^4 (r^2 - r_0^2)}{4 \rho_0 c^2 r_0^2 h} \]

\[ X_{a1} = \frac{2 \rho_0 cr_0 h}{\pi r^4} \left( -J_0 (kr_0) Y_1 (kr) + J_1 (kr_1) Y_0 (kr_0) \right) + \frac{2kr_0}{J_1 (kr_1) Y_1 (kr_0) + J_0 (kr_0) Y_1 (kr_1)} \frac{2kr_0}{(kr)^2 - (kr_0)^2} \]

\[ R_a = \frac{6 \mu}{\pi h^3} \beta \]

\[ \mu \ldots \text{dynamic viscosity} \]

\[ \beta \ldots \text{function of } (r_0/r) \]
Numerical Example

- **Disc Dimensions** $S = 100 \times 100 \, \mu m^2$ $t = 1.5 \, \mu m$ $h = 2 \, \mu m$

- **Air-Gap open: at the Circumference Side (side) / at the Centre (hole)**

- **Reactance $X_{a1}$**

- **Damping Resistance**
  - electrode open at the side:
    $R_{a(side)} = 1,1 \times 10^{12} \, [kg/m^4s]$
  - 1 hole 10x10 $\mu m^2$:
    $R_{a(hole)} = 6,5 \times 10^{12} \, [kg/m^4s]$
  - 25 holes 5x5 $\mu m^2$:
    $R_{a(hole)} = 7 \times 10^{10} \, [kg/m^4s]$
Application of the Model

Air-Coupled Arrangement

Transfer Function

(derived from the reciprocity theorem)

\[
\begin{pmatrix}
\frac{p_{out}}{i_{in}} \\
\frac{u_{in}}{q_{out}}
\end{pmatrix}_{q_{out} = 0} = \begin{pmatrix}
\frac{u_{in}}{q_{out}}
\end{pmatrix}_{i_{in} = 0}
\]
Sensor:

\[ M_s = \frac{u_{out}}{p_{in}} = \frac{U_0 S}{j\omega Z_m h} \quad [V/Pa] \]

Transmitter:

\[ M_t = \frac{p_{out}}{u_{in}} = \frac{j\pi \rho_0 f^2 M_s C_0}{d(1+jkR_{eff})} \quad [Pa/V] \]
Transfer Function:

\[ T \frac{u_{\text{out}}}{u_{\text{in}}} = \frac{\pi \rho_0 f^2 M M C t_0}{d (1 + jkR_{\text{eff}})} \]
Conclusion

• Complete mechanical/acoustic model is necessary to describe transducer behaviour

• The presented model can be easily implemented with electrical equivalent circuit

• The model can be used to determine
  – frequency response
  – input impedance
  – thermal noise due to transducer

• Future work
  – verification of the model with fabricated test structures
  – more detailed description of the acoustics part